

§1205 The Geometric Mean v. the Arithmetic Mean

The mean, or average, seems to be a simple concept we all learned in middle school. But because you, the attorney, are often dealing with economists who are well versed in all forms of math and statistics, you may encounter a report where the expert has used the geometric mean. Dr. Lawrence Spizman has written a brief explanation of the difference between the two types of means. He has used the strict formulas in his note, but you don't need to master these. Your own economist is paid to do that for you. There is one caution regarding the use of the geometric mean. Dr. Spizman has assumed that you will be dealing with a series of wage changes or medical cost changes that are always increasing. Should there be years in the series used by the economist where the change is zero or decreasing, then he shows the adjustment that can be made in order to use the geometric mean. Or you can simply use the arithmetic mean when there is zero or negative changes from year to year since the difference between the two is very small. Having said that, it is unlikely that you will encounter years where wages decreased and medical costs went down. The remainder of this section was written by Dr. Spizman. He has also included Table 38 to illustrate the differences.

GEOMETRIC v. ARITHMETIC MEAN WHICH ONE SHOULD THE FORENSIC ECONOMIST UTILIZE?

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Introduction

A Forensic Economist (FE) is often required to estimate growth rates for different elements of damages in personal injury or wrongful death cases. This short note only deals with two such growth rates: those involved with an individual's expected earnings (often wages) and those involved with the medical care components of an individual's life care plan. Ideally, historical earnings data is available to assist in estimating future expected earnings growth. In the absence of case-specific historical earnings data, general statistical averages from sources such as the *Consumer Price Index* (<http://data.bls.gov/PDQ/outside.jsp?survey=cu>) and the *Employment Cost Index* (<http://data.bls.gov/PDQ/outside.jsp?survey=ec>) can be utilized to determine historical averages. The *Employment Cost Index (ECI)* allows for tailoring industry and occupations specific to the plaintiff. The medical care component section of the *Consumer Price Index (CPI)* provides historical price changes that can be used to estimate future cost increases of the Life Care Plan.

Regardless of which historical index is used, the basis for estimating future wage or price growth rates is often based on historical averages. Section §1010 discusses time period for trends from historical data. The importance of determining growth rates, and thus the magnitude of the loss, is whether the arithmetic or geometric mean of

historical data is applied. Most readers are familiar with the arithmetic mean which simply adds a series of numbers together and divides that sum by the amount of observations you added together. However, if wage or price increases are added to the base, creating a cumulative or compounding effect from year to year, then the geometric mean (not arithmetic) of historical data should be utilized.

When to Utilize the Geometric Mean

To illustrate the difference in the two means, suppose a worker's wage increased 3.0 percent the first year, 5.0 percent the second year, and 4.0 percent the third year. The arithmetic mean is $(3.0 + 5.0 + 4.0) / 3 = 4.0$ percent. Notice the three numbers are added together to produce a sum and subsequently divided by 3.

The arithmetic mean can be illustrated by the following formula:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \frac{1}{n} (x_1 + \cdots + x_n).$$

However, if each wage increase is a permanent addition to the base wage (that is, cumulative rather than a one time lump sum amount) then the application of the geometric mean is required. In essence, we are compounding the growth of wages. The percentage increase in wages from year to year are not added to each year (as in the arithmetic mean) but instead are multiplied (or compounded) to get a product.

The geometric mean can be illustrated by the following:

$$\left(\prod_{i=1}^n a_i \right)^{1/n} = \sqrt[n]{a_1 \cdot a_2 \cdot \cdots \cdot a_n}$$

Applying the formula to our example above, note that the first year the worker's wages were multiplied by 1.03, the second year wages were multiplied by 1.05, and the third year they were multiplied by 1.04. This results in the geometric mean of as follows.

$$(1.03 * 1.05 * 1.04)^{1/3} = 3.91$$

The application of the geometric mean is similar to the compounding process of interest rates on a savings account. The geometric mean is used any time several quantities multiply together to produce a product. In the above example the arithmetic mean is 4.0 percent but the geometric mean is 3.91 percent. Because there is little variation from year to year, the difference is not that great. However, if the variation from year to year of percentage wage increases were greater, then the difference between the geometric and arithmetic mean would be more significant. For example, if the wage increase were 1.0 percent, 6.0 percent and 3.0 percent, then the arithmetic mean would amount to 3.33 percent $[(1 + 6 + 3) / 3 = 3.33]$ while the geometric mean amounts to 2.62 percent $[(1.01 * 1.06 * 1.03)^{1/3}]$, resulting in a 27.19 percent overestimation of the growth rate.

For positive growth rates, the geometric mean will always be less than or equal to the arithmetic mean. Thus, if we want to show the cumulative compounding effect of wage or price increases, the geometric mean should be applied so not to overestimate the loss. The reader can see the mathematical proof of this differential by doing a simple Google search of "Arithmetic-Geometric Mean Inequality".

Different components of life care plans also require increasing current costs into the future. The general medical care subcategories of the Consumer Price Index (M-

CPI) could be correlated to the life care plan. Simply select “Part 2” at the following site and locate Medical Care (<http://data.bls.gov/PDQ/outside.jsp?survey=cu>). Although the FE might have to defend matching the M-CPI category to the appropriate category within the life care plan, that is not at issue within this note. What is of importance is the fact that if the FE utilizes the historical arithmetic average of the appropriate M-CPI category instead of the geometric mean, then the total life care plan loss will be overestimated. Given the potentially large values of life care plans, especially for younger individuals, a small increase in the growth rate can raise future costs significantly.

Table 38 illustrates the five, ten, twenty, and thirty year differentials between the arithmetic and geometric means for the “All Items” component as well as various “Medical Care” components of the CPI from 1977 through 2006.

As previously noted, Table 38 demonstrates the difference between the arithmetic and geometric mean for the “All Items” component as well as various “Medical Care” components of the CPI. As exhibited in Table 38, the geometric mean is less than the arithmetic mean for each component that has positive historical growth. The geometric mean does not exist and therefore cannot be calculated for “Internal and Respiratory Over-the-Counter Drugs” (Column E) due to the negative growth (loss) rates in 2002, 2004, and 2005. Intuitively, there is no compound annual growth rate for a series of numbers that include negative growth (or loss). Since determining the geometric mean when a negative growth rate exists in a series of number is problematic, Column F in Table 38 illustrates how to approximate the geometric mean when there are non-positive numbers.

By way of example, at the end of 2001, the cost of “Internal and Respiratory Over-the-Counter Drugs” is 1.011 times greater than 2000. If the cost was \$15.00 in 2000, adjusted accordingly resulted in a cost of \$15.17 in 2001 ($\$15.00 * 1.011 = \15.165 , rounded to \$15.17). At the end of 2002, the cost would be 0.999 percent times the hourly wage of \$15.17 or \$15.15 ($15.17 * .999 = 15.15$). By applying the “corrective” geometric mean of 1.011 and 0.999 (two positive numbers), the problem of determining the geometric mean inclusive of negative growth is solved. The geometric mean for the years 2001 and 2002 is approximately 1.004982, which is less than the arithmetic mean of 1.005000. Thus, on average, the cost in the future can be multiplied by 1.004982 each year (an increase of earnings of .004982 per year), not .005 percent representing the arithmetic mean.

Column F transforms all data to positive numbers and illustrates that the geometric mean is less than the arithmetic mean for all positive numbers. Although the variation between the two means in Column F is very small that is not the situation with the other components. Table 38 shows the greater the variation of percentage changes from year to year, the larger the differential between the means. However, the magnitude of the differential is not the issue as much as the fact that the arithmetic mean will always overestimate losses.

Electronic technology today allows almost instantaneous access to government data. Thus, any additional time necessary to calculate the geometric mean is trivial given the more accurate measurement. A Forensic Economist would be hard-pressed not to have access to any of these data sources and quickly have an accurate estimate the geometric mean. Therefore, when cumulative compounding of growth rates is required, few limitations exist preventing a FE from making the correct geometric mean calculation.

Table 38: Arithmetic v. Geometric Mean

Year	A <i>CPI All goods and services</i>	B <i>Medical care</i>	C <i>Medical care commodities</i>	D <i>Pre-prescription drugs</i>	E <i>Internal and respiratory over-the-counter drugs</i>	F <i>Corrective Adjustment of Column E</i>	G <i>Medical care services</i>	H <i>Professional services</i>	I <i>Hospital and related services</i>	J <i>Nursing homes and adult daycare</i>
	percentage change from year to year									
1977	6.5	9.6	6.5	6.1	6.9	1.069	9.9	8.1		
1978	7.6	8.4	7.0	7.7	7.1	1.071	8.5	7.5		
1979	11.3	9.2	7.1	7.8	7.4	1.074	9.8	8.7	10.7	
1980	13.5	11.0	9.3	9.2	10.1	1.101	11.3	11.1	13.4	
1981	10.3	10.7	11.0	11.4	12.4	1.124	10.7	10.3	14.3	
1982	6.2	11.6	10.3	11.6	10.8	1.108	11.8	8.5	14.2	
1983	3.2	8.8	8.6	11.0	7.5	1.075	8.7	7.1	11.3	
1984	4.3	6.2	7.3	9.6	6.2	1.062	6.0	7.2	8.7	
1985	3.6	6.3	7.2	9.5	5.4	1.054	6.1	6.1	6.3	
1986	1.9	7.5	6.6	8.6	4.9	1.049	7.7	6.4	6.0	
1987	3.6	6.6	6.7	8.0	5.3	1.053	6.6	6.6	6.9	
1988	4.1	6.5	6.8	8.0	5.6	1.056	6.4	6.8	9.3	
1989	4.8	7.7	7.8	8.7	6.1	1.061	7.7	6.5	11.5	
1990	5.4	9.0	8.4	10.0	5.1	1.051	9.3	6.6	10.9	
1991	4.2	8.7	8.2	9.9	4.5	1.045	8.9	6.1	10.2	
1992	3.0	7.4	6.4	7.5	3.8	1.038	7.6	6.1	9.1	
1993	3.0	5.9	3.7	3.9	3.4	1.034	6.5	5.1	8.4	
1994	2.6	4.8	2.9	3.4	1.5	1.015	5.2	4.2	5.9	
1995	2.8	4.5	1.9	1.9	0.7	1.007	5.1	4.4	5.0	
1996	3.0	3.5	2.9	3.4	1.9	1.019	3.7	3.6	4.5	
1997	2.3	2.8	2.3	2.6	1.7	1.017	2.9	3.4	3.3	
1998	1.6	3.2	3.0	3.7	1.3	1.013	3.2	3.2	3.3	4.7
1999	2.2	3.5	4.0	5.7	0.3	1.003	3.4	3.2	4.2	4.2
2000	3.4	4.1	3.2	4.4	0.6	1.006	4.3	3.7	5.9	4.8
2001	2.8	4.6	4.0	5.4	1.1	1.011	4.8	3.7	6.6	4.1
2002	1.6	4.7	3.6	5.2	(0.1)	0.999	5.1	3.0	8.7	5.0
2003	2.3	4.0	2.5	3.1	1.3	1.013	4.5	2.9	7.3	5.7
2004	2.7	4.4	2.5	3.3	(0.2)	0.998	5.0	3.9	5.9	3.8
2005	3.4	4.2	2.5	3.5	(0.7)	0.993	4.8	3.8	5.3	3.3
2006	3.2	4.0	3.6	4.3	2.1	1.021	4.1	2.7	6.4	4.1
5 Yr Arithmetic	2.64	4.26	2.94	3.88	0.48	0.010048	4.70	3.26	6.72	4.38
5 Yr Geometric	2.55	4.25	2.89	3.81	n/a	0.010047	4.69	3.22	6.62	4.30
Difference	3.48%	0.19%	1.64%	1.89%		0.005419%	0.31%	1.13%	1.52%	1.93%
10 Yr Arithmetic	2.55	3.95	3.12	4.12	0.74	0.010074	4.21	3.35	5.69	
10 Yr Geometric	2.47	3.90	3.06	4.00	n/a	0.010074	4.14	3.33	5.44	
Difference	3.39%	1.18%	2.02%	3.05%		0.003650%	1.75%	0.71%	4.62%	
20 Yr Arithmetic	3.10	5.21	4.35	5.30	2.27	0.010227	5.46	4.48	6.93	
20 Yr Geometric	2.96	4.92	3.90	4.77	n/a	0.010224	5.18	4.27	6.51	
Difference	4.80%	5.71%	11.41%	11.04%		0.020325%	5.35%	4.84%	6.41%	
30 Yr Arithmetic	4.35	6.45	5.59	6.61	4.13	0.010413	6.65	5.68	7.98	
30 Yr Geometric	3.71	5.96	4.95	5.91	n/a	0.010408	6.19	5.25	7.38	
Difference	17.21%	8.12%	13.09%	11.86%		0.0530%	7.55%	8.16%	8.12%	

n/a: Not calculated due to negative numbers in series.